SPhT-00/167
BI-TP 2000/40
TPJU-15/2000
HD-THEP-00-62

QCD with Adjoint Scalars in 2D: Properties in the Colourless Scalar Sector

P. Bialas[1,a], A. Morel[2,b], B.Petersson[3,c], K. Petrov[3,d] and T. Reisz[2,4,e]

[1]Inst. of Comp. Science, Jagellonian University
33-072 Krakow, Poland

[2]Service de Physique Théorique de Saclay, CE-Saclay,
F-91191 Gif-sur-Yvette Cedex, France

[3]Fakultät für Physik, Universität Bielefeld
P.O.Box 100131, D-33501 Bielefeld, Germany

[4]Institut für Theor. Physik, Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg, Germany

Abstract

We present a numerical study of an SU(3) gauged 2D model for adjoint scalar fields, defined by dimensional reduction of pure gauge QCD in (2+1)D at high temperature. In the symmetric phase of its global Z_2 symmetry, two colourless boundstates, even and odd under Z_2, are identified. Their respective contributions (poles) in correlation functions of local composite operators A_n of degree $n = 2p$ and $2p + 1$ in the scalar fields ($p = 1, 2$) fulfill factorization. The contributions of two particle states (cuts) are detected. Their size agrees with estimates based on a meanfield-like decomposition of the $p = 2$ operators into polynomials in $p = 1$ operators. No sizable signal in any A_n correlation can be attributed to $1/n$ times a Debye screening length associated with n elementary fields. These results are quantitatively consistent with the picture of scalar "matter" fields confined within colourless boundstates whose residual "strong" interactions are very weak.

[a]pbialas@agrest.if.uj.edu.pl
[b]morel@spht.saclay.cea.fr
[c]bengt@physik.uni-bielefeld.de
[d]petrov@physik.uni-bielefeld.de
[e]t.reisz@thphys.uni-heidelberg.de

1 Introduction

Dimensional reduction is a powerful technique to study the infrared region of field theories at high temperature.[1]–[5] Combined with a non-perturbative lattice simulation of the reduced model, it has been employed to investigate the properties of gauge theories and QCD with dynamical quarks in the plasma phase [6]–[9]. For a recent review see [10]

It is, however, still not clear, what are the limitations and the domain of validity of this approach. Therefore, in a recent work [11], we have studied in detail the reduction to two dimensions of pure gauge QCD in (2+1) dimensions at high temperature. We refer the reader to this article for a more complete discussion of our motivations, references to the related literature and details on the reduction procedure. The reduced model is a model for scalars belonging to the SU(3) algebra (formerly the electric gluons in a static gauge). They interact with the 2D gauge fields, the static components of the original 3D magnetic gluons, and via an effective potential whose self-couplings were computed by perturbative integration over the non-static degrees of freedom. The main conclusion of [11], that dimensional reduction works very well in this case, followed from numerical simulations showing that the Polyakov loops correlations measured in (2+1)D QCD and in the reduced model were very close one to the other, over a wide range of temperatures, and down to quite short distances.

Less expected was the observation that these correlations assume a shape typical of *single* particle propagation, as opposed to the standard picture of a screening mass associated with *two* massive electric gluons. This feature, together with our previous motivations, invites us to pursue the numerical exploration of the reduced model per se.

While in [11] we only measured the correlations of Polyakov loops, here we analyze separately those of SU(3) invariant polynomials of degree n in the elementary scalars A, namely $A_n = \mathrm{tr}\, A^n$. Since the action is invariant under a global sign reversal of all the A's (Z_2-symmetry), we consider separately correlations involving even and odd polynomials, $n = 2p$ and $2p+1$. Using zero field initial conditions on large enough lattices, we make sure that we stay in the unbroken Z_2-phase, where even and odd operators do not mix. This is consistent with the perturbative way the model was derived, so that the properties found can be safely thought of as properties of the high temperature (2+1) QCD phase. The actual phase diagram relative to Z_2 will not be studied here. Related work on this symmetry for the $(3+1) \to 3$ reduction can be found in [9],[12]–[19].

The 2D action under consideration is recalled in Section 2, together with its meaning in terms of the (2+1)D QCD model, from which it originates, and the relevant operators and correlations are defined. The simulations are performed in the temperature range $2T_c$ to $12T_c$, where T_c is the deconfining temperature of the latter model. Our results are presented and discussed in Sections 3 and 4. In Section 3, we first show that the measured correlations fulfill the factorization properties expected if the lowest states in the $n-$even and $n-$odd channels are two distinct one particle states, whose masses are then extracted from the large distance decays. In Section 4, we further analyze the composite operators A_n, $n = 2p$ and $2p+1$ and their correlations $A_{n,m}$, showing that all the properties observed for $p = 2$ can be deduced with a good accuracy from their knowledge for $p = 1$. This follows from the assumption that, given the SU(3) and Z_2 symmetry constraints, the effective model for the elementary A's after integration over the gauge fields is a free field model for the massive composites A_2 and A_3. In particular, we give evidence that the (small) deviations from factorization observed at short distances are mainly due to intermediate states containing two of the above particles. The summary and the conclusions can be found in a last Section 5.

2 The Reduced Action, Operators and Correlations

In this section, we write down the reduced 2D action derived in [11], and define our notations and the quantities of interest for the present work.

The lattice is an $L_s \times L_s$ square; the spacing is a, set to one unless specified otherwise. The weight in the partition function is written $\exp(-S)$, with S a function of the SU(3) group elements $U(x;\mu)$, $\mu = 1,2$ on the links and of the scalars $A(x)$ in the adjoint representation on the sites:

$$A(x) = \sum_{\alpha=1}^{8} A^\alpha(x)\lambda^\alpha, \quad \mathrm{tr}\,\lambda^\alpha\lambda^\beta = \frac{1}{2}\delta_{\alpha\beta}. \tag{1}$$

Greek superscripts on A will always denote colour indices, unlike integers n, m used in powers of the algebra element A. We write the 2D reduced action as follows:

$$S = S_U + S_{U,A} + S_A, \tag{2}$$
$$S_U = \beta_3 L_0 \sum_x \Big(1 - \frac{1}{3}\Re\,\mathrm{tr}\,U(x;1)U(x+a\hat{1};2)U(x+a\hat{2};1)^{-1}U(x;2)^{-1}\Big),$$
$$S_{U,A} = \frac{\beta_3 L_0}{6}\sum_x\sum_{\mu=1}^{2}\mathrm{tr}\,\Big(D_\mu(U)A(x)\Big)^2, \tag{3}$$
$$D_\mu(U)A(x) = U(x;\mu)A(x+a\hat{\mu})U(x;\mu)^{-1} - A(x),$$
$$S_A = \sum_x k_2\,\mathrm{tr}\,A^2(x) + k_4\left(\mathrm{tr}\,A^2(x)\right)^2.$$

In the above, S_U is the pure gauge term, $S_{U,A}$ the gauge invariant kinetic term for the scalars and S_A the scalar potential, whose self couplings k_2 and k_4 result from the one-loop integration over the non-static components of the 3D gauge fields. All terms have a global Z_2-symmetry $A(x) \to -A(x)$, while the Z_3 symmetry of the original $(2+1)D$ $SU(3)$ model is broken by the perturbative reduction procedure. It was found in [11] that

$$\begin{aligned} k_2 &= -\frac{3}{2\pi}\Big(c_0 \log L_0 + c_1\Big); \quad c_0 = 1, \quad c_1 = \frac{5}{2}\log 2 - 1, \\ k_4 &= \frac{L_0^2}{64\pi}. \end{aligned} \tag{4}$$

The values of the parameters β_3, L_0 follow from the original lattice regularization of 3D pure QCD at temperature T and gauge coupling squared g_3^2 in the continuum. The latter has dimension one in energy and is used to set the scale:

$$\begin{aligned} \beta_3 &= \frac{6}{a g_3^2}, \\ L_0 &= \frac{1}{aT}. \end{aligned} \tag{5}$$

Accordingly, the continuum limit $a \to 0$ is obtained by letting β_3 and L_0 go to infinity with the dimensionless temperature

$$\tau = \frac{T}{g_3^2} = \frac{\beta_3}{6L_0} \tag{6}$$

being kept fixed.

We now turn to the definitions of the quantities relevant for the present investigation of the 2D model. The Polyakov loop operator

$$L(x) = \frac{1}{3}\mathrm{tr}\ \exp[i\, L_0\, A(x)] \tag{7}$$

is the order parameter for the global Z_3 symmetry of 3D pure QCD at finite temperature, known to be broken above the phase transition, which occurs at $\beta_3 \simeq 14.7$ for L_0=4 [20]. The reduced action, which is not Z_3 - invariant, is designed to reproduce the long distance physics in the broken Z_3-phase, and in [11] it was demonstrated that it does so very well from the numerical comparison of the Polyakov loop correlation $\mathcal{P}(x)$ in the full $(2+1)D$ $SU(3)$ theory and in the $2D$ reduced model:

$$\mathcal{P}(x) = \langle L(0)\, L^\dagger(x)\rangle - |\langle L\rangle|^2. \tag{8}$$

We now expand $L(x)$ in powers of $A(x)$, and we will study the operators $A^n(x)$ and correlations of their traces, defining

$$A_n(x) \equiv \mathrm{tr}\, A^n(x), \tag{9}$$

$$A_{n,m}(x) \equiv \langle A_n(x)\, A_m(0)\rangle - \langle A_n\rangle\, \langle A_m\rangle. \tag{10}$$

When not ambiguous, the notation A_n may also represent $\langle A_n(x)\rangle$. Any operator $A_n(x)$ is gauge invariant, scalar or pseudo-scalar under the Z_2-symmetry of S for n even or odd respectively. Because the reduced model was derived from a *small* A-fields expansion, its properties are significant for the (2+1)D model in the unbroken Z_2 phase only, where $A_{2p+1} = 0$ and A_{2p} is small.

We have performed a numerical simulation of the model defined above, with a flat measure for the A's and the standard De Haar measure for the gauge fields. The algorithm and error estimate techniques used are the same as in [11] and not reproduced here. The lattice size is $L_S = 32$, and $L_0 = 4$, throughout the present work. The β_3 values are 29 ($\sim$ twice the critical value for (2+1)D at $L_0 = 4$ [20]), 42, 84, and 173. We were able to extract information from operators and correlations corresponding to $n = 2$ to 5. For larger n, the signal/noise ratio becomes too small, the more so the temperature is high, which reduces the A_n's by a factor of order $\beta_3^{-n/2}$ (see (3)). We checked that the Polyakov loop correlations are actually fully reconstructed within errors by keeping $\{n, m\}$ up to $\{5, 5\}$ only in their decomposition into $A_{n,m}$'s obtained from the small A expansion of (8). The cases $n = 2$ and 3 for the 3D reduced model were investigated in [15].

3 The lowest states of the scalar spectrum

In this section we present our results for the $A_{n,m}$ correlations measured, and describe them for each β_3 in terms of two states of physical masses M_S and M_P, for n, m both even and both odd respectively.

In the simulations reported here, all runs were initialized with small A-fields values and the system stayed in the Z_2 unbroken phase, as desired, with $\langle A_{2p+1}(x)\rangle$ being always compatible with zero. From a few runs on smaller lattices however, we have indications that the stable phase, for the same values of the parameters, is in fact the broken phase, similarly to the situation encountered in ([12]-[15]).

For a first look at the data obtained in even and odd channels, we show in Fig. 1 the on-axis correlations $A_{n,m}(r)$, $n \leq m \in [2, 5]$ at $\beta_3 = 29$. In the even cases, the three correlations all have the same shape, and they decrease by about one order of magnitude each time two more powers of A are involved. The same is true for the odd cases, with a common decay of the correlations steeper than in the former case (smaller correlation length). The overall situation is identical for β_3 higher, although the accuracy of the measurements becomes less good at large r because the correlation lengths in lattice units increase with β_3.

Now we want to analyze these $A_{n,m}$ data quantitatively in terms of the lowest states of the spectrum. Let $m_i = a\,M_i$ be the lowest mass with quantum number $i = S, P$, in lattice units. For particle i we introduce a lattice propagator $\widetilde{\Delta}_{Latt}(m_i, p)$ in momentum space:

$$\begin{aligned} \widetilde{\Delta}^{-1}_{Latt}(m_i, p) &= \widehat{p}^2 + 4\sinh(m_i^2/4), \\ \widehat{p}^2 &= 4\sin^2(p_1/2) + 4\sin^2(p_2/2). \end{aligned} \tag{11}$$

The corresponding contribution to $A_{n,m}(r)$ then reads

$$A_{n,m}(m_i, r) = g^i_{n,m} \frac{1}{L_s^2} \sum_{p_1,p_2} \cos(p_1 r) \widetilde{\Delta}_{Latt}(m_i, p), \tag{12}$$

where $g^i_{n,m}$ measures the residue of $A_{n,m}$ at the pole of (11), which on large enough lattices sits at $p^2 \sim \widehat{p}^2 \sim -m_i^2$. With our definitions, $g^i_{n,m}$ is non zero only for $i = S$ if n and m are even, and for $i = P$ if n and m are odd. *Prior to any fit of the masses to the data*, we notice that a first consequence of our expectations on the lowest part of the spectrum comes from residue factorization $g^i_{n,m} = \gamma^i_n \gamma^i_m$, a property which we can probe directly on the correlations since, as r becomes large, it implies

$$X_n \equiv \frac{A_{n,n}(r)\, A_{n+2,n+2}(r)}{A^2_{n,n+2}(r)} \to 1. \tag{13}$$

That it is so is demonstrated on Figs. (2-5) showing X_2 and X_3 (symbols $\diamond$) for $\beta_3 = 29$ and 84. The agreement is very good in all cases, although the quality of the data is poorer for X_3 due to the correlations involving A_5 getting very small. Similar results are obtained for other values of β_3. We thus conclude at this point that single particle propagation accounts very well for the largest correlation length occuring in each of the two channels. Most of the observed deviations of X_n from one will be interpreted in the next section in terms of two particle state contributions (symbols $\circ$ in the same figures).

We now proceed to assign values to the two lowest masses M_S and M_P expected from the above findings. This we do by various ways, in order to further enforce the statement that the correlations do have the characteristics associated with the pole structure of Eq. (11). Down to $r \sim 1$, an excellent approximation to the on-axis correlation (12) is given by

$$A_{n,m}(m_i, r) \simeq c \left(\frac{1}{[m_i r]^{1/2}} e^{-m_i\, r} + \frac{1}{[m_i(L_s - r)]^{1/2}} e^{-m_i(L_s - r)} \right), \tag{14}$$

where c is constant in r. We performed fits of this formula to all our $A_{n,m}(r)$ data taken at $r > r_{min}$. These fits are stable with respect to r_{min} provided it is larger than about 4, and the values found for m_i in different correlations are always consistent with each other. The smallest errors were obtained by using fits to $A_{2,2}$ and $A_{3,3}$.

Effective masses can also be obtained without any fitting by using $0-$momentum correlations, defined for a generic $x-$space correlation $C(x_1, x_2)$ by

$$C^0(r) = \frac{1}{L_s} \sum_{x_2} C(r, x_2). \tag{15}$$

If the lowest mass in C is m, the ansatz (11) gives

$$C^0(r) \propto \cosh\Big(m(L_s/2 - r)\Big), \tag{16}$$

and m can be extracted at any r by inverting this relation:

$$m = \log\Big(Y(r) + \sqrt{Y^2(r) - 1}\Big), \tag{17}$$
$$Y(r) = \frac{C^0(r+1) + C^0(r-1)}{2C^0(r)}.$$

As an overall consistency check, we have extracted an effective mass $m^{eff}(r)$ from $0-$momentum Polyakov loops correlations (8), and compared it to the m_S values obtained by our fits to $A_{2,2}$. We find that $m^{eff}(r)$ is indeed nearly constant, in fact slowly decreasing towards a value compatible with m_S, due to smaller and steeper contributions to (8) of the heavier particle P.

A contrario, we invalidate the interpretation of the largest correlation length in $A_{n,n}$ as being n times shorter than the "Debye screening length", the inverse of a mass m_E associated with "electric" gluons of the initial (2+1)D model (the scalars of the reduced model). This scenario was advocated by D'Hoker in his perturbative study of QCD_3 at high temperature [21]. If such was the case, the on-axis correlations $A_{n,n}$ should rather look like

$$A_{n,n}(n\, m_E, r) \propto \left(\frac{1}{[m_E r]^{1/2}} e^{-m_E\, r} + \frac{1}{[m_E(L_s - r)]^{1/2}} e^{-m_E(L_s - r)} \right)^n, \tag{18}$$

which differs in shape from (14), as was illustrated in [11] for Polyakov loop correlations. We nevertheless tried fits with (18), but got definitely worse agreement, in the range of temperatures, which we have investigated, i.e up to $12T_c$. Hence the above picture is ruled out by the data in this temperature range, and if a mass can be defined for the electric gluon in high temperature QCD_3 it is most probably larger than both $m_S/2$ and $m_P/3$. In a "constituent gluon" picture, as advocated in Ref. [22], one would have bound states instead of a cut. One would, however, expect $m_P/m_S \approx 3/2$.

β_3	m_S	m_P
29	0.317(6)	0.57(3)
42	0.28(1)	0.57(3)
84	0.252(6)	0.43(2)
173	0.197(7)	0.32(1)

Table 1: Masses in lattice units for the S and P states, as measured from fits to $A_{2,2}$ and $A_{3,3}$ respectively, as functions of β_3.

Our final results for the S and P masses in lattice units are collected in Table 1 as functions of β_3. They are fitted to $A_{2,2}$ and $A_{3,3}$ respectively. The values for m_S agree with those obtained in [11] from the Polyakov loop correlations.The ratio m_P/m_S fluctuates between 1.6 and 2 as β_3 varies. Within the accuracy reached presently, it is not possible to assess any definite behaviour for it.

4 Weak "Strong" Interactions Between Colourless States

Here we will show that even at quite short distances (compared to m_S^{-1}) all the condensates $A_n \equiv \langle A_n(x) \rangle$ and correlations $A_{n,m}(r)$ can be reconstructed to a good accuracy from the data for A_2, $A_{2,2}$ and $A_{3,3}$. The assumption is that the elementary fields $A^\alpha(x)$ (Greek superscripts are colour indices) interact only through S and P exchanges between the non-interacting composite $A_2(x)$ and $A_3(x)$, the scale of the fields being fixed by the size of A_2, while A_3=0. The precise way how this idea is implemented and the corresponding technicalities are detailed in the appendix.

Here we limit ourselves to the simplest applications and give the results, starting with the local condensates.

4.1 Weak Residual Interactions: The A-fields condensates

Since SU(3) has rank 2, any $A_n(x)$ can be reduced to a polynomial in $A_2(x)$ and $A_3(x)$. An elegant method [23] and explicit formulae are given in the Appendix. For n odd A_n is zero by Z_2 symmetry. For n even, we apply Wick contraction to all pairs of A^α elementary fields, followed by the meanfield-like substitution

$$A^\alpha(x)\, A^\beta(x) \;\to\; \frac{1}{4}\delta_{\alpha,\beta}\, A_2(x). \tag{19}$$

As an illustration, consider A_4. With the definitions of Section 2, we have

$$2A_4(x) = A_2^2(x) = \frac{1}{2^2} \sum_{\alpha,\beta=1}^{8} A^\alpha(x) A^\alpha(x) A^\beta(x) A^\beta(x). \tag{20}$$

There we apply (19) and then replace $A_2(x)$ by its average A_2. The $A^\alpha A^\alpha$ and $A^\beta A^\beta$ contractions give $(8 \times A_2/4)^2$, and the additional contributions from $\alpha = \beta$ give $2 \times 8(A_2/4)^2$. Noting that $A_{2,2}(0) = \langle A_2^2(x) \rangle - A_2^2$ (see (10)), the net result can be put into the two equivalent forms

$$\begin{aligned} 2A_4 &= \frac{5}{4} A_2^2, && (21) \\ A_{2,2}(0) &= \frac{1}{4} A_2^2. && (22) \end{aligned}$$

This prediction is remarkably well verified in all cases. At $\beta_3 = 29$, the left and right hand sides of (22) are respectively $4.86(1)\,10^{-3}$ and $4.825(10)\,10^{-3}$. They are $6.093(13)\,10^{-4}$ and $6.069(1)\,10^{-4}$ at $\beta_3 = 84$. Similar manipulations lead to

$$A_{3,3}(0) = \frac{5}{64} A_2^3. \tag{23}$$

In this case the left and right hand sides are measured to be $2.105(7)\,10^{-4}$ and $2.095(6)\,10^{-4}$ for $\beta_3 = 29$, $9.365(30)\,10^{-6}$ and $9.344(3)\,10^{-6}$ for $\beta_3 = 84$. Hence the effects of residual interactions via non quadratic effective couplings in $A_2(x)$ and $A_3(x)$ are less than the percent in the correlations at zero distance.

4.2 Weak Residual Interactions: Properties of the Correlations

As we have seen in section 2 (see Fig. 1), the different $A_{n,m}(r)$'s corresponding to the same channel have very similar shapes in r. Their analysis in terms of one particle exchange was successful, confirmed by the agreement with residue factorization. Nevertheless although the quantity X_n of Eq. (13) does go to one at large r, it is significantly different from one at medium and short distances (see Figs. 2-5). We will now show that two particle exchange is responsible for most of this lack of factorization.

The simplest consequence of our assumptions for correlations at finite r is, using (21),

$$A_{2,4}(r) = \frac{5}{4} A_2\, A_{2,2}(r), \tag{24}$$

which is very well verified for any r as shown in Fig.6 for $\beta_3 = 29$. A similar agreement is found for the relation

$$A_{3,5}(r) = \frac{35}{24} A_2 \, A_{3,3}(r), \tag{25}$$

derived in the appendix. A new situation arises when we consider $A_{4,4,}$ or $A_{5,5}$ where both the initial and final states may couple to a two particle state, $(S\,S)$ or $(S\,P)$ respectively. Then the intermediate state in a connected correlation between 0 and r may consist of either one or two particles. For example, to compute $A_{4,4,}(r)$, we apply the substitution rule (19) to the sum (20), and then average using the definitions of $A_{2,2}$ and A_2. One finds

$$4A_{4,4}(r) = \left(\frac{5}{4}\right)^2 \left[4A_2^2 \, A_{2,2}(r) \, + 2 \, A_{2,2}^2(r)\right]. \tag{26}$$

A similar treatment given in the appendix leads to the prediction

$$A_{5,5}(r) = \left(\frac{35}{24}\right)^2 \left[A_2^2 \, A_{3,3}(r) \, + \, A_{2,2}(r) \, A_{3,3}(r)\right]. \tag{27}$$

In the two expressions above, the second contribution, a product of two propagators in space, is that of a two-particle intermediate state, and it provides a correction to exact factorization. From the definitions Eq.(13) of X_2 and X_3, one actually gets the following estimates:

$$X_2(r) \simeq \widetilde{X}_2(r) \equiv 1 + \frac{A_{2,2}(r)}{2A_2^2(r)}, \tag{28}$$

$$X_3(r) \simeq \widetilde{X}_3(r) \equiv 1 + \frac{A_{2,2}(r)}{A_2^2(r)}. \tag{29}$$

The estimates $\widetilde{X}_2(r)$, $\widetilde{X}_3(r)$ are displayed in Figs. 2,3 (resp. 4,5), for comparison with the measured values $X_2(r)$, $X_3(r)$ at $\beta_3 = 29$ (resp. 84). We see that the corrections to factorization implied by two-particle propagation provide a reasonable explanation of the behaviour observed for the X's at intermediate and short distances. This is especially true in the case of X_2, showing that there is very little room for contributions from direct non-quadratic couplings in $A_2(x)$ in the full effective action (that resulting from integration over the gauge fields). This justifies our statement that the residual interactions between the colourless boundstates of the adjoint scalars are very weak.

5 Conclusions

In this paper, we have studied properties of the two dimensional model derived in [11] by dimensional reduction of 3D QCD at high temperature. In this model, scalar fields A in the adjoint representation of SU(3) interacts via SU(3) gauge fields U, in addition to a self-interaction generated by integration over the non-static 3D gauge degrees of freedom. Such properties are interesting since it was shown in [11] that dimensional reduction works remarkably well in this case. Also, they offer an opportunity to explore non-perturbative features in a low dimensional situation where the IR singularities are particularly severe.

By means of numerical simulations, we explored that part of phase space where the Z_2-symmetry $A \to -A$ is unbroken, in accordance with the small A expansion used to derive the model, known to be valid quite soon above the transition temperature of pure 3D QCD. We identified two boundstates S and P, respectively even and odd under Z_2, and thus coupled to monomials respectively of degree $2n$ and $2n+1$ in the A's. The S signal coincides with that previously obtained from Polyakov loops correlations [11], where however the P-state contributions could not be disentangled.

These results came out from the measurement of three even-even and three odd-odd distinct correlations, as functions of the on-axis distance r. Great care was taken in the analysis of their shape in r, with the result that in all cases, the signal found was that expected from the occurrence of genuine poles in momentum space. A contrario, this demonstrates that the picture where the decay with r of such correlations reflects the propagation in 3D of $p = 2n$ or $2n+1$ "electric gluons", i.e. a correlation length equal to $1/p$ times the "Debye screening length", is inadequate in the case under study.

By comparing the size of the three different correlations measured for each of the S and P sectors, we were able to show that residue factorization holds, as expected on general grounds when one particle propagates between different states. The agreement with factorization was expectedly found to be particularly good at large distances, but we could even show that deviations at shorter distances are to a large extent compatible with propagation of two particles, namely two S or S and P respectively in the S or P channel. The overall picture thus is that the scalar sector of the reduced model at large distances, thought to accurately describe static properties of 3D QCD at high temperature, consists of two weakly interacting particles, respectively even and odd under the Z_2 symmetry of the model.

There are several problems, which this study invites to investigate further. Of course, similarly detailed analysis for full QCD in (3+1)D would be interesting. Also, there is clearly a need to find correlation functions, whose behaviour would unambiguously define an analog to the QED Debye screening length in non abelian cases. Finally, the construction of a reduced model where the Z_3-symmetry of the pure gauge theory is not spoiled by the reduction process is highly desirable [24], with the hope that it exhibits a transition to a symmetric Z_3 phase analogous to the low temperature QCD phase.

6 Acknowledgments

We thank the DFG for support under the contract Ka 1198/4-1. K.P. was also supported by DAAD and P.B partially by KBN grant P03B01917.

Appendix: Mean Field Technique for Composite Fields Correlators

A Formulae for Traces and Determinants

Let ϕ be a complex $N \times N$ matrix. Here we give an elegant trick [23] to compute the traces

$$\phi_n \equiv \mathrm{tr}\, \phi^n \tag{30}$$

for $n > N$ given ϕ_p for $p \leq N$.

Consider the determinant $P_N(t) \equiv Det(1 - t\phi)$, where t is a complex variable. It is a polynomial of degree N in t and its term of degree N is $(-1)^N Det(\phi)$, and we have

$$\log\Big(P_N(t)\Big) \;=\; \mathrm{tr}\; \log\left(1 - t\phi\right). \tag{31}$$

Both sides of this identity can be expanded in t in some finite neighbourhood of zero. The method consists in identifying the coefficients of the two series. The N first orders determine the coefficients of $P_N(t)$ from the ϕ_n's ,$n \leq N$. Then, the higher orders directly express any ϕ_n, $n > N$ as a function of the ϕ_p's, $p \leq N$. Note that instead of computing $Det(\phi)$ from the order N coefficient of P_N, one can alternatively compute ϕ_N given $Det(\phi)$, which is convenient for SU(N) group matrices.

If applied to $\phi = A$, an element of the SU(3) algebra, (in which case $A_1 = 0$), this technique gives A_n, $n > 3$ in terms of A_2 and A_3 taken as independent variables. The first non trivial identities are

$$A_4 \;=\; \frac{1}{2}(A_2)^2,$$

$$
\begin{aligned}
A_5 &= \frac{5}{6} A_2 A_3, \\
A_6 &= \frac{1}{4}(A_2)^3 + \frac{1}{3}(A_3)^2, \\
A_7 &= \frac{7}{12} A_3 (A_2)^2, \\
A_8 &= \frac{1}{8}(A_2)^4 + \frac{4}{9}(A_3)^2 A_2,
\end{aligned}
\tag{32}
$$

and the determinant is

$$
Det A = \frac{2A_3}{3!}. \tag{33}
$$

In what follows, we will have to manipulate monomials of the elementary scalar fields $A^\alpha(x)$, defined for SU(N) through

$$
A(x) = \sum_{\alpha=1}^{N^2-1} A^\alpha(x) \lambda^\alpha \tag{34}
$$

where the traceless basis λ^α is subject to the normalization

$$
\mathrm{tr}\, \lambda^\alpha \lambda^\beta = \frac{1}{2} \delta_{\alpha\beta}. \tag{35}
$$

On this basis the anti-commutators read

$$
\{\lambda^\alpha, \lambda^\beta\} = c_{\alpha\beta}\, 1_N + \sum_{\gamma=1}^{N^2-1} d_{\alpha\beta\gamma}\, \lambda^\gamma, \tag{36}
$$

with real and totally symmetric tensors c and d. With these normalizations and notations, we have

$$
\mathrm{tr}\, A^2 = \sum_{\alpha\beta=1}^{N^2-1} \mathrm{tr}\, [\lambda^\alpha \lambda^\beta]\, A^\alpha A^\beta = \frac{1}{2} \sum_{\alpha\beta=1}^{N^2-1} \delta_{\alpha\beta}\, A^\alpha A^\beta, \tag{37}
$$

$$
\mathrm{tr}\, A^3 = \sum_{\alpha\beta\gamma=1}^{N^2-1} \mathrm{tr}\, [\lambda^\alpha \lambda^\beta \lambda^\gamma]\, A^\alpha A^\beta A^\gamma = \frac{1}{4} \sum_{\alpha\beta\gamma=1}^{N^2-1} d_{\alpha\beta\gamma}\, A^\alpha A^\beta A^\gamma. \tag{38}
$$

The projection property

$$
\sum_{\alpha=1}^{N^2-1} \lambda^\alpha_{ab} \lambda^\alpha_{dc} = \frac{1}{2}\left(\delta_{ac}\delta_{bd} - \frac{1}{N}\delta_{ab}\delta_{cd}\right) \tag{39}
$$

can be used to derive that for any pair X, Y of complex $N \times N$ matrices the following identities hold:

$$\sum_\alpha \mathrm{tr}\,\lambda^\alpha X\ \mathrm{tr}\,\lambda^\alpha Y \;=\; \frac{1}{2}\left(\mathrm{tr}\,XY \;-\; \frac{1}{N}\,\mathrm{tr}\,X\ \mathrm{tr}\,Y\right), \tag{40}$$

$$\sum_\alpha \mathrm{tr}\,X\lambda^\alpha Y\lambda^\alpha \;=\; \frac{1}{2}\left(\mathrm{tr}\,X\ \mathrm{tr}\,Y \;-\; \frac{1}{N}\,\mathrm{tr}\,XY\right). \tag{41}$$

In what follows we specialize to $N = 3$.

B Correlators of Composite Operators

By gauge invariance, $A_2(x)$ and $A_3(x)$ can be chosen as the two effective degrees of freedom. By Z_2-symmetry, the even and odd sectors under $A \to -A$ decouple. Here we derive consequences of the assumption that their dynamics is determined at leading order by their given vacuum expectation values A_2 and 0 respectively and their connected two-body correlations $A_{2,2}(x)$ and $A_{3,3}(x)$.

We use a Wick like treatment to express the higher order connected correlation functions and averages through the quantities mentioned above. If $n = 2p$, any $A^\alpha(x)$ is assigned to belong to a pair $A_2(x)$, then considered as a free field denoted $S(x)$. So each monomial is replaced by a sum over all such pairings, and each of the p pairs is subject to the substitution W_2,

$$W_2: \qquad A^\alpha A^\beta \to \delta_{\alpha\beta}\,\frac{1}{4}\,S(x), \tag{42}$$

leading to a monomial of degree p in $S(x)$. If $n = 2p+1$, one first performs the $p-1$ possible substitutions W_2 (the result after p substitutions would transform as an octet under $SU(3)$ and thus vanishes), which yield a monomial necessarily proportional to $S^{p-1}(x)\,\mathrm{tr}\,A^3(x)$. There we apply the substitution W_3,

$$W_3: \qquad \mathrm{tr}\,A^3(x) \to P(x), \tag{43}$$

where $P(x)$ is also considered as a free field.

Once the local operators have been expressed in terms of $S(x)$ and $P(x)$, any average is obtained by using

$$\langle S(x)\rangle = A_2, \tag{44}$$

$$\langle S(x)\,S(0)\rangle = A_{2,2}(x) + A_2^2, \tag{45}$$

$$\langle P(x)\,S(0)\rangle = 0, \tag{46}$$

$$\langle P(x)\rangle = 0, \tag{47}$$

$$\langle P(x)\,P(0)\rangle = A_{3,3}(x). \tag{48}$$

These rules generalize the way how in Section 4 we computed averages involving $A_4(x)$. Let us now detail calculations involving $A_5(x)$.

From Eqs.(32, 37, 38), we have

$$\frac{6}{5} A_5(x) = \sum_{\alpha\,\beta} A^\alpha A^\beta \,\mathrm{tr}\, \lambda^\alpha \lambda^\beta \sum_{\gamma,\delta,\epsilon} A^\gamma A^\delta A^\epsilon \,\mathrm{tr}\, \lambda^\gamma \lambda^\delta \lambda^\epsilon. \tag{49}$$

We apply rule W_2 to the right hand side. The contraction of α with β produces $S(x)\,P(x)$ once. Using Eqs. (40, 41, 37, 38), one finds that the 6 W_2-contractions of either α or β with either one of the three other indices contribute each the same amount

$$\frac{1}{4} S(x) \times \frac{1}{2} \,\mathrm{tr}\, A^3(x), \tag{50}$$

that is from rule W_3

$$\frac{1}{8} S(x)\, P(x). \tag{51}$$

We thus arrive to the substitution

$$A_5(x) \to \frac{5}{6}\,(1 + \frac{6}{8})\, S(x) P(x) = \frac{35}{24}\, S(x) P(x), \tag{52}$$

which we perform in the two point correlations $A_{3,5}(x) \equiv \langle A_3(x)\, A_5(0)\rangle$ and $A_{5,5}(x) \equiv \langle A_5(x)\, A_5(0)\rangle$ to get

$$A_{3,5}(x) = \frac{35}{24} \langle P(x)\, P(0)\, S(0)\rangle, \tag{53}$$

$$A_{5,5}(x) = \left(\frac{35}{24}\right)^2 \langle P(x)\, P(0)\, S(x)\, S(0)\rangle. \tag{54}$$

According to Eqs. (44-48), these averages are given by

$$A_{3,5}(x) = \frac{35}{24} A_2\, A_{3,3}(x), \tag{55}$$

$$A_{5,5}(x) = \left(\frac{35}{24}\right)^2 A_{3,3}(x) \left(A_2^2 + A_{2,2}(x)\right). \tag{56}$$

As a last application, we derive the value of $A_{3,3}(0) \equiv \langle A_3^2(x)\rangle$. By definition

$$A_3^2(x) = \sum_{\alpha\,\beta\,\gamma} A^\alpha A^\beta A^\gamma \,\mathrm{tr}\, \lambda^\alpha \lambda^\beta \lambda^\gamma \sum_{\delta\,\epsilon\,\zeta} A^\delta A^\epsilon A^\zeta \,\mathrm{tr}\, \lambda^\delta \lambda^\epsilon \lambda^\zeta, \tag{57}$$

where all the fields are taken at the same point x. Applying all possible W_2 substitutions and using Eqs. (40, 41) leads to the substitution

$$A_3^2(x) \to \frac{5 S^3(x)}{64}, \tag{58}$$

and averaging via Eq. (44) provides the final result (23).

References

[1] P. Ginsparg, *Nucl. Phys.* **B170** (1980) 388.

[2] T. Appelquist and R. Pisarski, *Phys. Rev.* **D23** (1981) 2305.

[3] S. Nadkarni, *Phys. Rev.* **D27** (1983) 917; *Phys. Rev.* **D38** (1988) 3287.

[4] N. P. Landsman, *Nucl. Phys.* **B322** (1989) 498.

[5] T. Reisz, *Z. f. Phys.* **C53** (1992) 169.

[6] P. Lacock, D. E. Miller and T. Reisz, *Nucl. Phys.* **B369** (1992) 501.

[7] L. Kärkkäinen, P. Lacock, D.E. Miller, B. Petersson and T. Reisz, *Phys. Lett.* **B282** (1992) 121.

[8] L. Kärkkäinen, P. Lacock, B. Petersson and T. Reisz, *Nucl. Phys.* **B395** (1993) 733.

[9] K. Kajantie, M. Laine, K. Rummukainen, M. Shaposhnikov, *Nucl. Phys.* **B503** (1997) 357.

[10] O. Philipsen, "Static correlation lengths in QCD at high temperature and finite density", hep-lat/0011019

[11] P. Bialas, A. Morel, B. Petersson, K. Petrov and T. Reisz, "High Temperature 3D QCD: Dimensional Reduction at Work", *Nucl. Phys.* **B581** (2000) 477.

[12] L. Kärkkäinen, P. Lacock, D.E. Miller, B. Petersson and T. Reisz, *Nucl. Phys.* **B418** (1994) 3.

[13] T. Reisz, "Dimensionally Reduced SU(2) Yang-Mills Theory is Confined",in *Quantum Field Theoretical Aspects of High Energy Physics*, 230-235, B. Geyer and E.M. Ilgenfritz Eds., Frankenhausen 1993.

[14] K. Kajantie, M. Laine, J. Peisa, A. Rajantie, K. Rummukainen and M. Shaposnikov, *Phys. Rev. Lett.* **79** (1997) 3130.

[15] K. Kajantie, M. Laine, A. Rajantie, K. Rummukainen, and M. Tsypin, *JHEP* **9811** (1998) 11.

[16] F. Karsch, M. Oevers and P. Petreczky, *Phys. Lett.***B442** (1998) 291.

[17] S. Datta and S. Gupta, *Nucl. Phys.* **B534** (1998) 392 *Phys. Lett.* **B471** (2000) 382.

[18] A. Hart, O. Philipsen, J.D. Stack and M. Teper, *Phys. Lett.* **B396** (1997) 217.

[19] A. Hart and O. Philipsen, *Nucl. Phys.* **B572** (2000) 243.

[20] C.Legeland, *PhD. Thesis, "Aspects of (2+1) Dimensional Lattice Gauge Theory"* (University of Bielefeld, Germany, September 1998).

[21] E. D'Hoker, *Nucl. Phys* **B201** (1982) 401.

[22] W. Buchmuller and O. Philipsen *Phys. Lett.* **B397** (1997) 112.

[23] We thank M. Bauer for providing us with this simple trick.

[24] R. Pisarski, "Quark-Gluon Plasma as a condensate of $SU(3)$ Wilson Lines", hep-ph/0006205. K. Kajantie, M. Laine, J. Peisa, A. Rajantie, K. Rummukainen, M. Shaposhnikov *Phys. Rev. Lett.* **79** (1997) 3130.

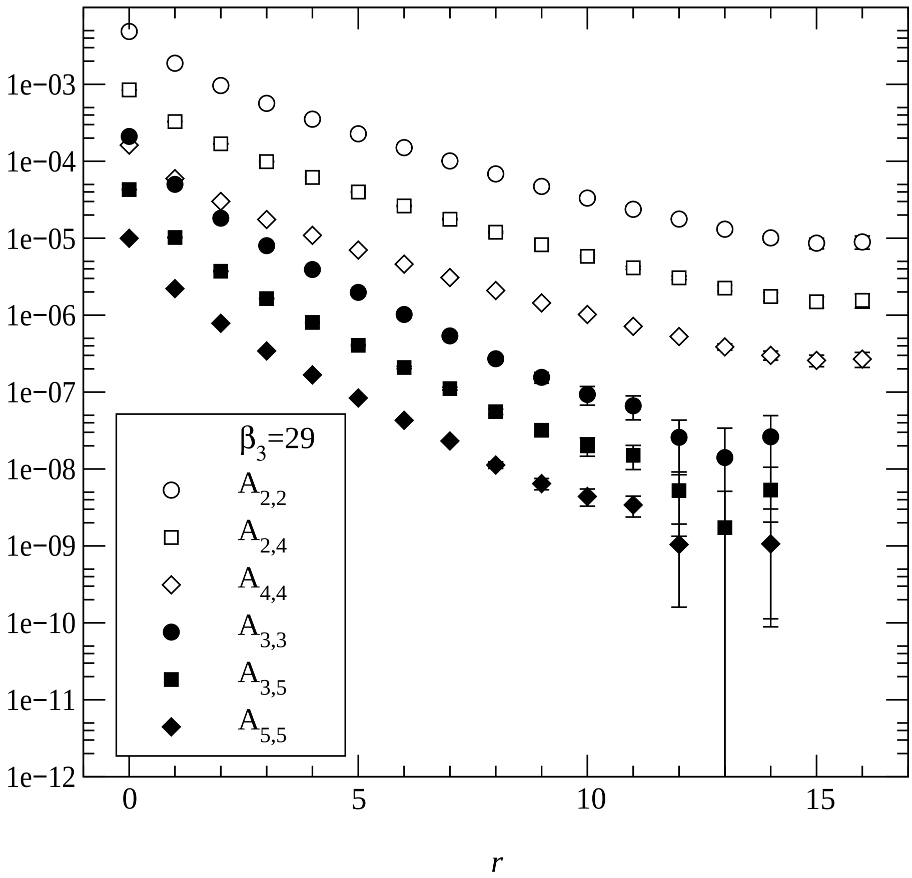

Figure 1: The on-axis correlations $A_{n,m}(r)$ at $\beta_3 = 29$. The even cases $[n,m] = [2,2],[2,4]$ and $[4,4]$ all have the same shape, and the odd cases $[3,3],[3,5]$ and $[5,5]$, again similar with each other in shape, are steeper.

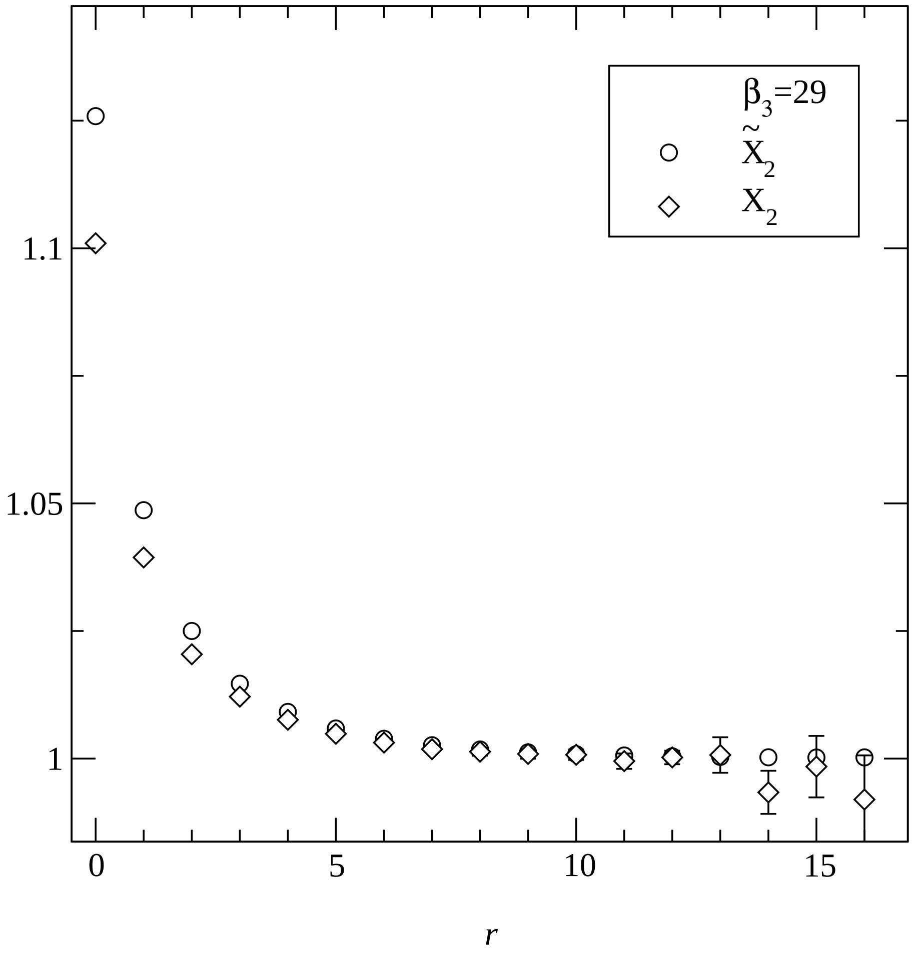

Figure 2: Residue factorization: data for the quantity X_2, Eq. (13) at $\beta_3 = 29$. It approaches one at large r. The quantity $\widetilde{X}_2$ corresponds to our interpretation (Section 4, Eq. (28)) of the deviation from one of X_2 at shorter distances.

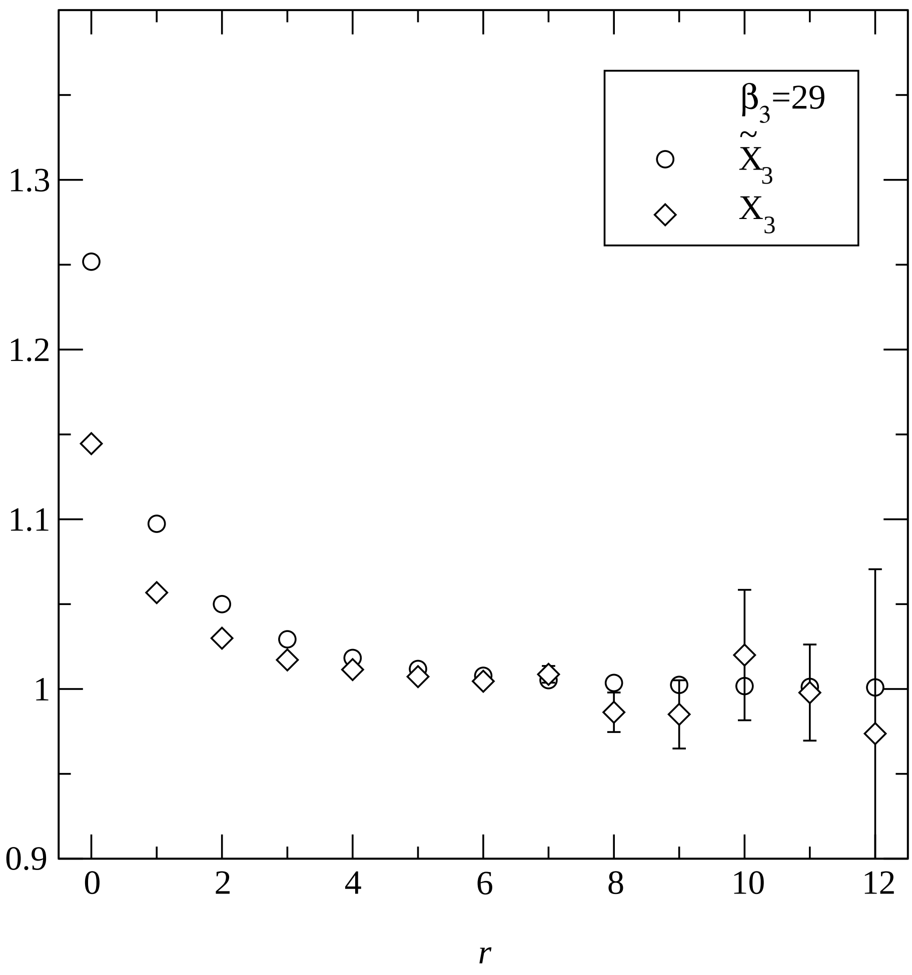

Figure 3: Residue factorization: data for the quantity X_3, Eq. (13) at $\beta_3 = 29$. It approaches one at large r. The quantity $\widetilde{X}_3$ corresponds to our interpretation (Section 4, Eq. (29)) of the deviation from one of X_3 at shorter distances.

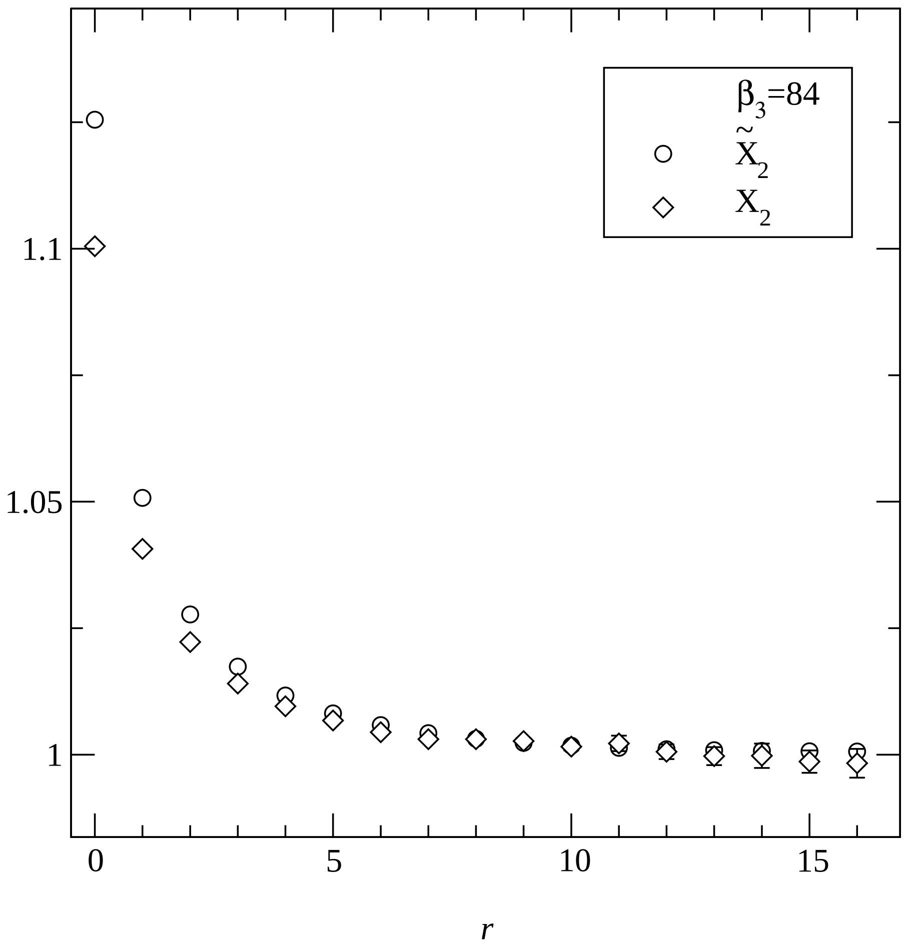

Figure 4: Same as in Fig. 2 at $\beta_3 = 84$.

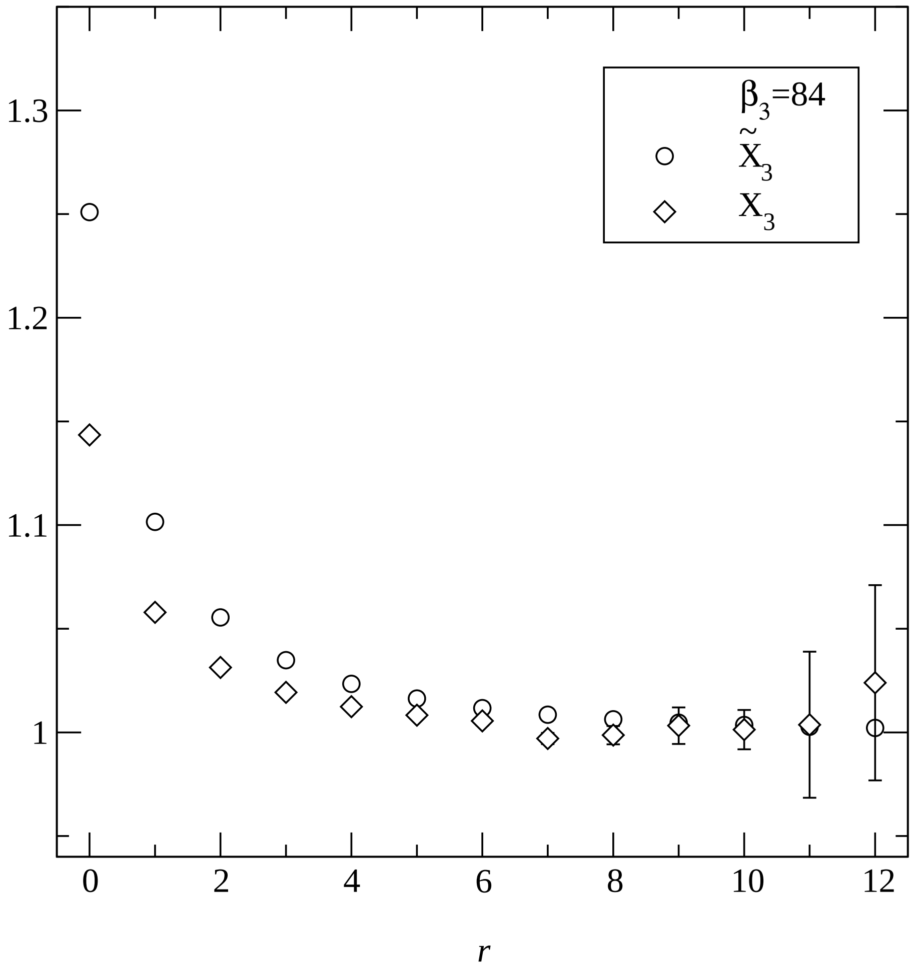

Figure 5: Same as in Fig. 3 at $\beta_3 = 84$.

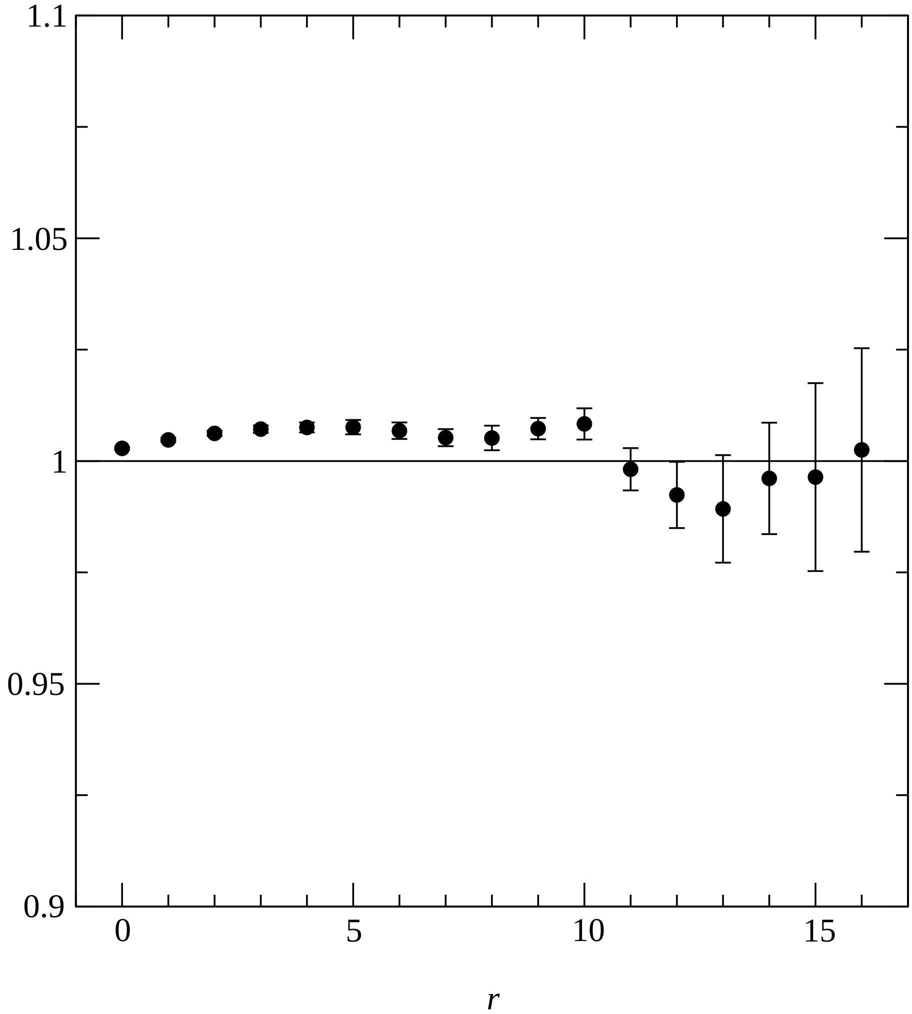

Figure 6: Plot of $4A_{24}/5A_2A_{22}$ at $\beta_3 = 29$, which is one if Eq. (24) holds.